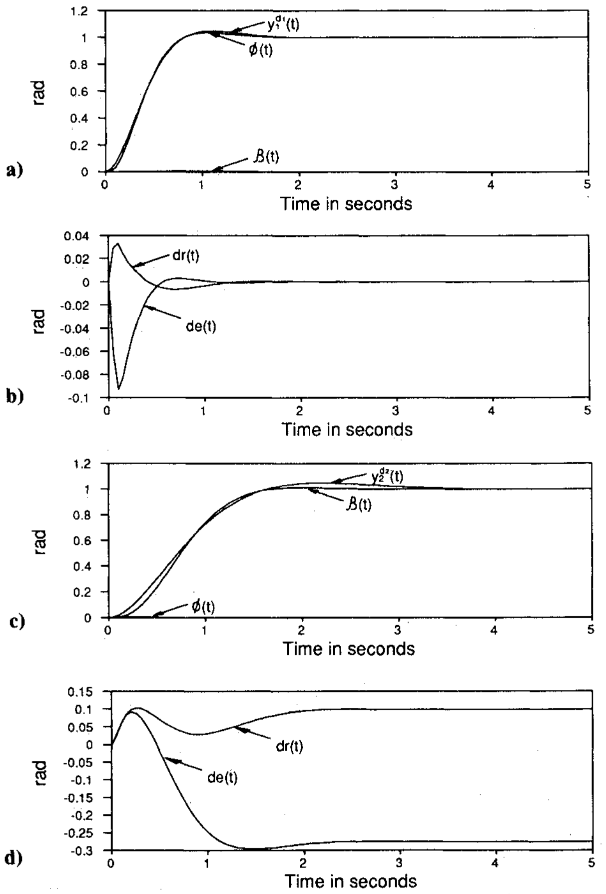

Fig. 2 Reference command $(\phi_c(t), \beta_c(t)) = (1\ \text{rad}, 0)$: a) controlled variables and b) control surface deflections; reference command $(\phi_c(t), \beta_c(t)) = (0, 1\ \text{rad})$: c) controlled variables and d) control surface deflections.

specifications are approximately met. The roll-angle tracking maneuver with approximately zero sideslip shown in Fig. 2a is practically used in RPV flight control. A PC-AT compatible computer requires approximately 60 min to obtain the preceding solution. By including numeric and step-response graphics output at each iteration, the proposed design procedure can provide useful visual information on how the actual step response of the system converges to the desired step response.

IV. Conclusion

This Note has proposed a synthesis methodology for automating the design of linear optimal control systems such that practical performance specifications are met in a satisfactory manner. Decoupled command tracking specifications in the time domain have been considered at present and a multivariable proportional plus integral control structure has been employed. The proposed design procedure has been applied in the design of a decoupled lateral control system for an RPV. The results obtained, part of which are presented in Fig. 2, demonstrate the effectiveness of the proposed design method.

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Model Reduction for Systems with Integrators

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Introduction

MODEL order reduction methods for stable linear systems are based on joint controllability and observability tests through balancing of system grammians.[1] For systems with integrators, grammians do not exist; thus, model reduction based on its grammian properties cannot be executed. Note, however, that systems with integrators are controllable and observable; hence, these properties can still be used for model reduction. In this Note the reduction algorithm for systems with integrators is derived.

Reduction

Consider a linear system with the state-space representation (A, B, C), with p inputs, q outputs, and n states. It is called a system with integrators, if it is observable and controllable, has $n - m$ poles stable, has the remaining m poles at zero, and has a nondefective A (geometric multiplicity of poles at zero is m). The reduction problem for systems with integrators is solved by introducing antigrammians.

For a controllable and observable triple (A, B, C) the matrices V_c, V_o satisfying the following Riccati equations:

$$V_cA + A^TV_c + V_cBB^TV_c = 0$$
$$V_oA^T + AV_o + V_oC^TCV_o = 0 \tag{1a}$$

are the controllability and observability antigrammians. For stable controllable and observable systems, $V_c = W_c^{-1}$, $V_o = W_o^{-1}$, where W_c, W_o are controllability and observability grammians satisfying the Lyapunov equations

$$AW_c + W_cA^T + BB^T = 0, \qquad W_oA + A^TW_o + C^TC = 0 \tag{1b}$$

The grammians for a system with integrators do not exist, but antigrammians do; for an unobservable or uncontrollable sys-

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tem antigrammians do not exist, but grammians do. The existence of antigrammians is exploited for the balancing and model reduction of systems with integrators.

For a stable, controllable, observable, and balanced system the grammians as well as the antigrammians are equal and diagonal $W_c = W_o = \Gamma$, $V_c = V_o = \Pi$, and $\Pi = \Gamma^{-1}$, where $\Gamma = \operatorname{diag}(\gamma_i)$, $\Pi = \operatorname{diag}(\pi_i)$, $i = 1, 2, \ldots, n$, and satisfy the following equations.

$$\Pi A_b + A_b^T \Pi + \Pi B_b B_b^T \Pi = 0, \qquad \Pi A_b^T + A_b \Pi + \Pi C_b^T C_b \Pi = 0 \tag{2a}$$

$$A_b \Gamma + \Gamma A_b^T + B_b B_b^T = 0, \qquad \Gamma A_b + A_b^T \Gamma + C_b^T C_b = 0 \tag{2b}$$

A balanced system with integrators (A_b, B_b, C_b) has antigrammians that are in the form $\Pi = \operatorname{diag}(0_m \;\; \Pi_o)$, where 0_m is a $m \times m$ zero matrix, and A_b is block diagonal, $A_b = \operatorname{diag}(0_m \;\; A_{bo})$. To prove it, consider A, B, and C in the form

$$A = \operatorname{diag}(0_m \;\; A_o), \qquad B^T = [B_r^T \;\; B_o^T], \qquad C = [C_r \;\; C_o] \tag{3}$$

Matrix A in the form of Eq. (3) always exist due to m poles at zero, and B, C exist due to nondefectiveness of A. From Eq. (1a) it follows that

$$V_{crr} B_r = 0, \qquad C_r V_{orr} = 0, \qquad V_{cro} = 0, \qquad V_{oro} = 0 \tag{4}$$

where V_c, V_o are divided conformably to A:

$$V_c = \begin{bmatrix} V_{crr} & V_{cro} \\ V_{cro}^T & V_{coo} \end{bmatrix}, \qquad V_o = \begin{bmatrix} V_{orr} & V_{oro} \\ V_{oro}^T & V_{ooo} \end{bmatrix}$$

For a controllable and observable system the matrices B_r and C_r are of full rank, thus, it follows from Eq. (4) that $V_{crr} = 0$, and $V_{orr} = 0$, and that $V_c = \operatorname{diag}(0_m \;\; V_{coo})$, and $V_o = \operatorname{diag}(0_m \;\; V_{ooo})$, which in balanced coordinates gives $\Pi = \operatorname{diag}(0_m, \Pi_o)$.

A balanced representation $A_b = T_b^{-1} A T_b$, $B_b = T_b^{-1} B$, and $C_b = C T_b$ of a system with integrators is obtained by the transformation T_b:

$$T_b = T_1 T_2 \tag{5}$$

The transformation T_1 turns A into the block diagonal form $A_1 = \operatorname{diag}(0_m, A_o)$, e.g., into real modal form, $A_1 = T_1^{-1} A T_1$; the transformation T_2 is in the form $T_2 = \operatorname{diag}(I_m, T_{bo})$, where I_m is an identity matrix of order m, and T_{bo} balances A_o, $A_{bo} = T_{bo}^{-1} A_o T_{bo}$.

Assume that the antigrammian Π of the balanced system is ordered increasingly, $\Pi = \operatorname{diag}(\pi_1, \pi_2, \ldots, \pi_{n-1}, \pi_n)$, where $\pi_i \geq 0$, $\pi_{i+1} \geq \pi_i$, $i = 1, \ldots, n$, with the first m singular values at zero, $\pi_i = 0$, $i = 1, \ldots, m$. In this case the system is reduced by truncating the last $n - k$ states of the balanced representation and leaving its first k states. Let the matrices A_b, B_b, and C_b be partitioned conformably:

$$A_b = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \qquad B_b = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \qquad C_b = [C_1 \;\; C_2] \tag{6}$$

then the reduced system representation (A_r, B_r, C_r, D_r) is $A_r = A_{11}$, $B_r = B_1$, $C_r = C_1$.

Examples

A system with integrators with the state-space representation

$$A = \begin{bmatrix} -5 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0.5 & 5 & -20 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 40 & 40 & -0.2 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B^T = [1 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0], \qquad C = [0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 10]$$

has one pole at zero. Although the system is controllable and observable, its controllability and observability grammians do not exist. The balanced representation is obtained with the transformation T_b [see Eq. (5)]

$$A_b = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.1063 & -0.0549 & -0.0527 & 0.0091 & 0.0011 \\ 0 & -0.0549 & -0.1055 & -0.9949 & 0.0324 & 0.0040 \\ 0 & 0.0527 & 0.9949 & -0.8882 & 0.0328 & 0.0040 \\ 0 & 0.0091 & 0.0324 & -0.0328 & -5.1058 & -1.2717 \\ 0 & 0.0011 & 0.0040 & -0.0040 & -1.2517 & -1.9794 \end{bmatrix}$$

$$B_b^T = [20 \;\; 13.9642 \;\; 3.8892 \;\; -3.2384 \;\; -0.5972 \;\; -0.0735]$$

$$C_b = [9.9964 \;\; -13.9642 \;\; -3.8892 \;\; -3.2384 \;\; 0.5972 \;\; -0.0735]$$

and the balanced antigrammian

$$\Pi = \operatorname{diag}(0, 0.0011, 0.0139, 0.0169, 28.6333, 7330.82)$$

satisfies Eq. (2a). Its last two (largest) singular values indicate that the system can be reduced to the system of order 4. Indeed, the output of the reduced model, obtained by truncation, is very close to the output of the full-order model, as illustrated by the magnitude of the transfer functions of the full and the reduced order models in Fig. 1.

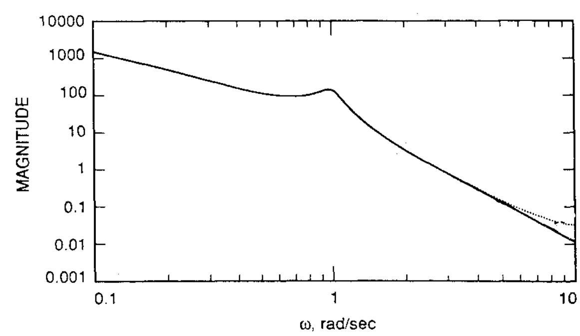

Fig. 1 Magnitude and phase of transfer function of the full-order and reduced system with integrators.

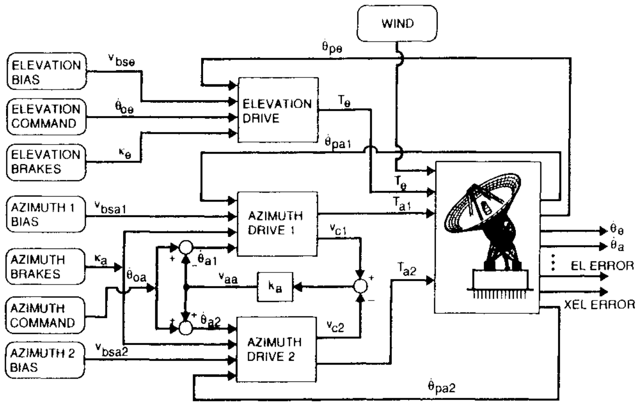

Fig. 2 Rate loop control system of the DSS-13 antenna.

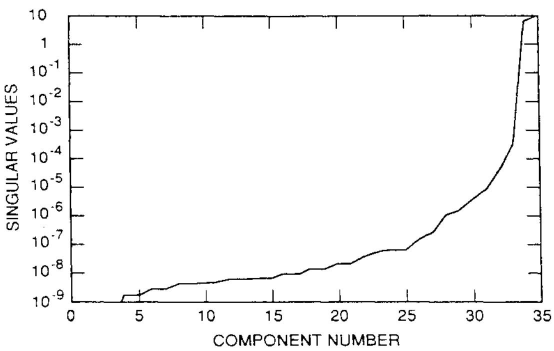

Fig. 3 Singular values of the balanced antigrammians of the rate loop model.

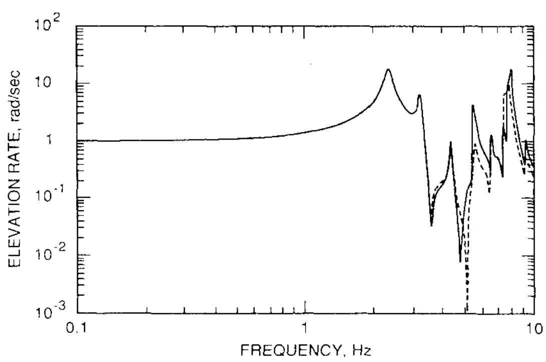

Fig. 4 Magnitude of the elevation rate transfer function (rad/s) for the full and the reduced rate loop model.

Consider the NASA Deep Space Network DSS-13 antenna rate loop model (see Fig. 2; for details of the model see Ref. 2). It consists of the antenna structural model and models of elevation and azimuth drives. The 35-state rate loop model has three poles at zero and is reduced while using balanced antigrammians; their singular values are shown in Fig. 3. By deleting the states with the largest singular values, the model has shrunk from 90 states to 27 states. The reduced model preserves the full-model properties, as illustrated by the frequency-response plots in Fig. 4.

Conclusions

Antigrammians, similarly to grammians, reflect controllability and observability properties of a system. Unlike grammians, antigrammians do exist for systems with integrators, hence making their reduction possible. In this Note the reduction algorithm for systems with integrators is derived. The algorithm is based on balancing the antigrammians and has the same computational effectiveness as the regular balancing procedure. It was applied to the reduction of the NASA Deep Space Network antenna model.[2] As a result, the 90-state model has been reduced to a 27-state model, which preserves the full-model properties.

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Minimizing Selective Availability Error on Satellite and Ground Global Positioning System Measurements

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Introduction

THE global positioning system (GPS) will turn on selective availability (SA) encryption, on a regular basis, to degrade the positioning accuracy for users denied access to such encryption. The SA reduces the user positioning accuracy in two ways. First, artificial offsets are added onto the broadcast GPS ephemerides and, second, the GPS clocks that generate the carrier phase and coded signals are dithered. For non-real-time users, the first aspect is of no concern. The effects of the SA clock dithering may in principle be removed by differencing between receivers observing the same GPS satellites. However, this is possible only if the receiver clocks are sufficiently good to keep the time-tags accurate to better than 1 ms. Although most ground receiver clocks can be synchronized to high accuracy, nonsimultaneity as large as 20 ms exists due to the unequal light-time between a given GPS satellite and different receivers. For receivers onboard a user satellite, the nonsimultaneity may be far greater. For instance, the U.S./French ocean topography experiment satellite, Topex/Poseidon,[1] which will be in orbit during 1992–1995, will carry a GPS receiver driven by a free-running crystal clock. Although the clock will be monitored from time to time using onboard real-time navigation, it will not be realigned to GPS clocks so that continuity in carrier phase can be maintained. Hence, the time-tags on the measurements will be drifting away from the correct time. The clock information will be recorded and telemetered back to ground, together with the tracking data.

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